OSCILLATIONS OF AN ICE SHEET UNDER A PERIODICALLY VARYING LOAD

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Interest in ice sheet dynamics has been stimulated by its many practical applications. Among these are problems of increasing the efficiency of the relatively recently discovered resonant method of ice-breaking using air cushion vessels (ACV's) [1].

The factors affecting flexure-gravitation wave (FGW) parameters, and thus, the ice-breaking ability of an ACV for uniform rectilinear motion of the latter were considered in [2]. As large scale model and field experiments with ACV's have shown, the capabilities of the resonant method can be improved. If the dimensions of the body of water or the vessel parameters do not permit excitation of waves of sufficient intensity to destroy the ice during uniform rectilinear motion, the ice cover can be broken by an additional dynamic load produced by periodic change of the pressure in the air cushion. The vertical oscillations of the vessel produced in this manner at some resonant frequency ω lead to excitation of a flexure-gravitation wave which destroys the ice over a significant area. The character of the ice damage is shown in Figure 1.

This present work will study the dependence of the stress-deformed state of the ice sheet upon propagation of waves stemming from a periodically varying load with rectangular form. The problem of ice sheet oscillations due to action of different types of loads has been studied in many works. Results of a study of transient oscillations of an ice sheet, treated as an elastic plate, under the action of moving harmonically varying pressure were presented in [3]. In [4] the process of development of transient flexure-gravitation waves produced by impulsive disturbances caused by displacement of the bottom of the basin were studied. The solution of the planar problem of the effect of periodic disturbances on oscillations of elastic ice with and without consideration of drift were obtained in [5, 6].

Analysis of known experimental-theoretical studies of plate dynamics on an elastic base indicate how well this problem has been studied. However the question of calculating the stress-deformed state of an ice sheet under conditions of flexure-gravitation resonance remains unconsidered. The present study will attempt to solve this problem.

1. Formulation of the Problem and Theoretical Solution. Numerous experiments have shown that in ice sheet loading by resonant flexure gravitation waves the ice manifests viscoelastic properties. Therefore in solving the problem both the plastic and elastic properties of the ice will be considered.

The maximum frequency of ACV vertical oscillation which can be realized in practice is much less than the quantity 2c/h (where c is the speed of transverse elastic waves in the ice sheet and h is the ice thickness), permitting use of the theory of inflection of thin plates [6]. Transient flexure-gravitation oscillations of the ice sheet for forced vertical harmonic oscillations of the vessel will be studied in the linear formulation. The ice will be considered a viscoelastic plate of infinite extent, the behavior of which is described by the Kelvin–Foight model.

The differential equation for inflection of the ice sheet has the form

$$D\left(1+\tau\frac{\partial}{\partial t}\right)\nabla^4 w+\rho_1 h\frac{\partial^2 w}{\partial t^2}+\rho_2\frac{\partial\Phi}{\partial t}\Big|_{z=0}=f(x,y,t), \tag{1.1}$$

where $D = Gh^3/3$ is the cylindrical strength of the plate; G is the modulus of elasticity for shear; τ is the deformation relaxation time; w is the deflection of the ice; ρ_1, ρ_2 are the densities of ice and water respectively; f(x, y, z) is the load distributed over the ice surface; x, y, z is a Cartesian coordinate system with z axis directed vertically upward; Φ is the potential of liquid motion velocity, satisfying the equation

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$$\nabla^2\Phi = 0. \tag{1.2}$$

The boundary conditions for Eq. (1.2) are Eq. (1.1) and the equality

$$\partial w/\partial t = \partial\Phi/\partial z|_{z=0},\ \partial\Phi/\partial z|_{z=-H}$$

(H = const = water depth). The initial conditions $\Phi(x, y, z, 0) = 0$, $w(x, y, 0) = 0$ express the absence of disturbances in the ice—water system.

Since the solution for an arbitrary load can be obtained by superposition, we will first consider the action of a concentrated load

$$f(x, y, t) = P\delta(x, y)\theta(t).$$

Here $P = P_0\exp(i\omega t)$; P_0 = const; $\delta(x, y)$ is a delta function; $\theta(t)$ is a Heaviside function: $\theta(t) = \begin{cases}1, t \geqslant 0,\\ 0, t < 0;\end{cases}$ ω is the circular frequency of the forced oscillations.

The solution of Eq. (1.2) with consideration of boundary conditions for plate deflections and the velocity potential can be written in the form

$$w(x, y, t) = \frac{P_0}{(2\pi)^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} w_{\lambda\eta}(t)\exp[-i(\lambda x + \eta y)]\,d\lambda\,d\eta,$$

$$\Phi(x, y, t) = \frac{P_0}{(2\pi)^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \Phi_{\lambda\eta}(t)\operatorname{ch}[(H + z)k]\exp[-i(\lambda x + \eta y)]\,d\lambda\,d\eta.$$

The delta function is represented here as an integral

$$\delta(x, y) = \frac{1}{(2\pi)^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\exp[-i(\lambda x + \eta y)]\,d\lambda\,d\eta.$$

According to Eq. (1.1) the functions $w_{\lambda\eta}(t)$ satisfy the equation

$$\left(\rho_1 h + \frac{\rho_2}{k\operatorname{th} kH}\right)\ddot{w}_{\lambda\eta} + D\tau k^4\dot{w}_{\lambda\eta} + (Dk^4 + \rho_2 g)\,w_{\lambda\eta} = \exp(pt)\,\theta(t) \tag{1.3}$$

and the initial conditions $w_{\lambda\eta}(0) = \dot{w}_{\lambda\eta}(0) = 0$, where $k = \sqrt{\lambda^2 + \eta^2}$, $p = i\omega$. From the boundary conditions we also have

$$\Phi_{\lambda\eta}(t) = \frac{1}{k\operatorname{sh} kH}\dot{w}_{\lambda\eta},$$

which can be used to determine $\Phi(x, y, t)$.

The solution of Eq. (1.3) can be found by the operation method. We denote by $\bar{w}_{\lambda\eta}(\bar{p})$ the Laplace transform of the function $w_{\lambda\eta}(t)$:

$$L(w_{\lambda\eta}(t)) = \int\limits_0^{\infty}\exp(-\bar{p}t)\,w_{\lambda\eta}(t)\,dt = \bar{w}_{\lambda\eta}(\bar{p}).$$

We then obtain

$$L(\exp(pt)\theta(t)) = 1/(\bar{p} - i\omega),\quad L(\dot{w}_{\lambda\eta}(t)) = \bar{p}\bar{w}_{\lambda\eta}(\bar{p}),\quad L(\ddot{w}_{\lambda\eta}(t)) = \bar{p}^2 w_{\lambda\eta}(\bar{p});$$
$$w_{\lambda\eta}(\bar{p}) = \frac{n}{(\bar{p} - i\omega)(\bar{p}^2 + l\bar{p} + m)} \tag{1.4}$$
$$(n = 1/(\rho_1 h + \rho_2/(k\operatorname{th} kH)),\quad l = D\tau k^4 n,\quad m = (Dk^4 + \rho_2 g)n).$$

Using inversion formulas with Eq. (1.4) for $t \geq 0$ we find

$$w_{\lambda\eta}(t) = n\frac{(k_2 - p)\exp(k_1 t) - (k_1 - p)\exp(k_2 t) + (k_1 - k_2)\exp(pt)}{(k_1 - k_2)(p^2 + lp + m)}$$

$$\left(k_1 = -\frac{l}{2} + \sqrt{\left(\frac{l}{2}\right)^2 - m}, \quad k_2 = -\frac{l}{2} - \sqrt{\left(\frac{l}{2}\right)^2 - m}\right).$$

Now let the load distributed over the area Ω of the plane x, y have the form

$$f(x, y, t) = rq(x, y)\exp(pt)\theta(t).$$

Here $r > 0$ is the "amplitude" factor, g(x, y) is the intensity of the load, related to the vessel weight by the expression

$$Q = \iint_{\Omega} |q(x, y)|\, dx\, dy.$$

Then

$$w(x, y, t) = \frac{r}{(2\pi)^2}\,\mathrm{Real}\iint_{\Omega} q(\xi, \zeta)\, d\xi\, d\zeta \times \tag{1.5}$$
$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w_{\lambda\eta}(t)\exp\{-i[\lambda(x-\xi) + \eta(y-\zeta)]\}\, d\lambda\, d\eta.$$

Assuming that q(x, y) = const, while the load is distributed over the area of a rectangle Ω with sides 2a, 2b ($-a \leq x \leq a$, $-b \leq y \leq b$), from Eq. (1.5) we obtain

$$w(x, y, t) = \frac{rq}{\pi^2}\,\mathrm{Real}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\sin\lambda a}{\lambda}\,\frac{\sin\eta b}{\eta}\, w_{\lambda\eta}(t)\exp[-i(\lambda x + \eta y)]\, d\lambda d\eta.$$

From the known deflection w we determine the maximum stresses produced from the expressions

$$\sigma_x = \frac{6M_x}{h^2}, \quad \sigma_y = \frac{6M_y}{h^2}, \quad \tau_{xy} = \frac{6M_{xy}}{h^2},$$

where $M_x = -D\left(1 + \tau\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right)$; $M_y = -D\left(1 + \tau\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right)$; $M_{xy} = D(1-\mu)\left(1 + \tau\frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial x \partial y}$; μ is the Poisson coefficient for the ice.

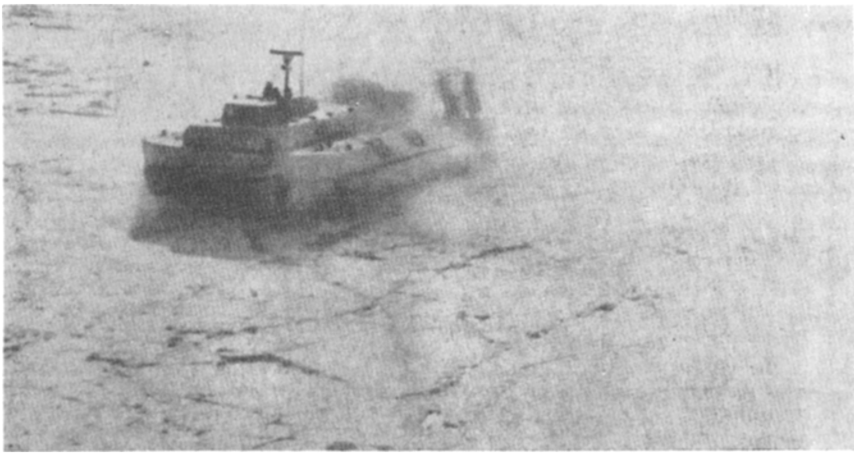

Fig. 1

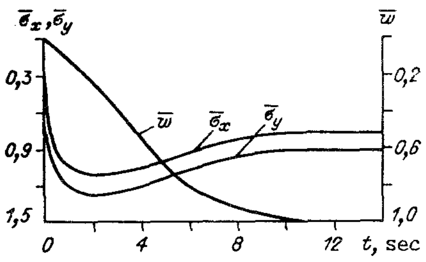

Fig. 2

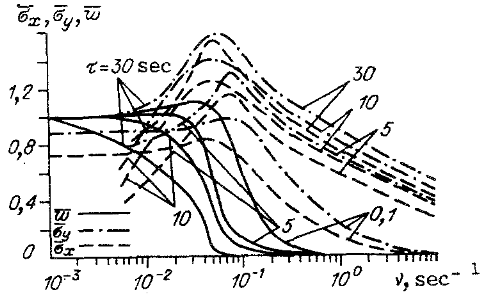

Fig. 3

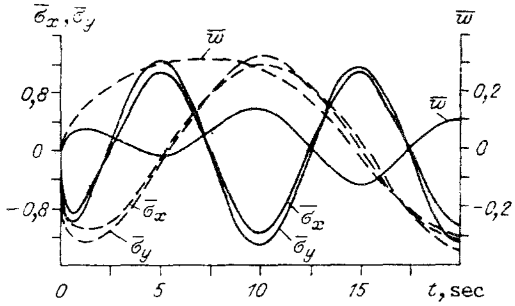

Fig. 4

The deflection of the ice sheet w and the bending moments M_x, M_y, M_{xy}, can be represented as the sums of two terms, the first of which is the result of the forced system oscillations only, while the second is a transient which decays with time:

$$w = B(A_1 \exp(pt) + A_2),\ M_x = BD(A_3 \exp(pt) + A_4),$$
$$M_y = BD(A_5 \exp(pt) + A_6),\ M_{xy} = BD(1 - \mu)(A_7 \exp(pt) + A_8).$$

Here $B = \frac{4qr}{\pi^2}$; $A_i = \int_0^\infty\int_0^\infty C_i \cos\lambda x \cos\eta y\, d\lambda\, d\eta$, $\quad i = 1, 2, \ldots, 8$; $\quad C_1 = \frac{n}{p^2 + lp + m}\frac{\sin\lambda a}{\lambda}\frac{\sin\eta b}{\eta}$;

$$C_2 = \frac{C_1}{k_1 - k_2}[(k_2 - p)\exp(k_1 t) - (k_1 - p)\exp(k_2 t)] \quad \text{for } k_1 \neq k_2;$$
$$C_2 = C_1[(k_1 - p)t - 1]\exp(k_1 t) \text{ for } k_1 = k_2;$$
$$C_3 = (\lambda^2 + \mu\eta^2)(1 + \tau p)C_1;\ C_4 = (\lambda^2 + \mu\eta^2)(C_2 + \tau\dot C_2);$$
$$C_5 = (\eta^2 + \mu\lambda^2)(1 + \tau p)C_1;\ C_6 = (\eta^2 + \mu\lambda^2)(C_2 + \tau\dot C_2);$$
$$C_7 = (1 + \tau p)C_1\lambda\eta \operatorname{tg}\lambda x \operatorname{tg}\eta y;\ C_8 = (C_2 + \tau\dot C_2)\lambda\eta \operatorname{tg}\lambda x \operatorname{tg}\eta y.$$

We note that the transient processes for free system oscillations, i.e., for f = 0, are also defined by the roots k_1 and k_2. This process is of a decaying oscillatory character for

$$4\pi^2\nu_c^2 = m - \left(\frac{l}{2}\right) > 0 \qquad (1.6)$$

(ν_c is the frequency of free system oscillation).

2. Results. The expressions presented above were used to calculate the stress-deformed state of an ice sheet for various frequencies ν of forced oscillation and time τ. Calculations were performed for μ = 0.33, h = 0.5 m, H = 5 m, ρ_1 = 900 kg/m^3, a = 10 m, ρ_2 = 1000 kg/m^3, G = $2\cdot10^9$ Pa, b = 5 m, q = −2000 Pa, r = 1, τ = 10 sec. As the characteristic deflection the static value w_0 = 0.044 m was chosen at x = y = 0, with a characteristic stress σ_0 = 10^6 Pa.

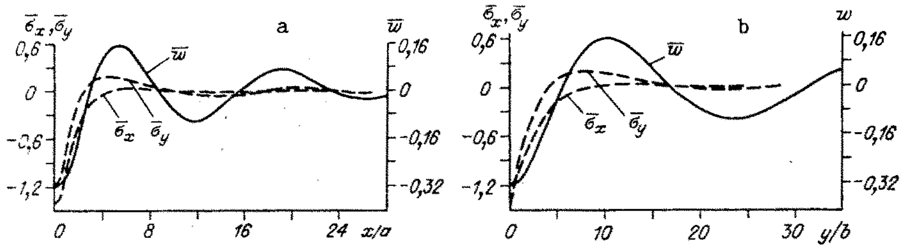

Fig. 5

Figure 2 shows results of calculations of $\bar{w} = w/w_0$, $\bar{\sigma}_x = \sigma_x/\sigma_0$ and $\bar{\sigma}_y = \sigma_y/\sigma_0$ at the point x = y = 0 for sudden load application ($\nu = 0$). Such ice sheet loading is characterized by significant (up to 30%) growth in stress and gradual increase in ice deflection. Stabilization of the process occurs over a time approximately equal to the relaxation time τ.

Calculations show that the stress-deformed state of the ice sheet depends significantly on τ, i.e., the parameter characterizing the viscous properties of the ice. Different τ values lead to a qualitative change in the character of deformation, as well as differences in the quantitative resonant frequency values (Fig. 3). However, as follows from processing of recordings of damping ice sheet oscillations [7], the value of τ for natural ice is quite stable and lies in the range 10-15 sec. Therefore the most probable resonant frequency for the calculated ice configuration $\nu_P = 0.05$ sec^{-1}. This value practically coincides with the resonant frequency of a flexure-gravitation wave excited by a load moving over the ice. The maximum stresses and deformations of the ice sheet correspond to forced oscillation frequencies in the range of ice—water system free oscillations defined by Eq. (1.6).

The development of the process of exit to a regime of forced ice oscillations about the static equilibrium position of the water—ice—load system is shown in Fig. 4 for $\nu = 0.1$ and 0.05 sec^{-1} (solid line and dashes). As follows from the calculations, the transient process decays over approximately two forced oscillation periods.

The decay of the wave as a function of distance from the center of load application for $\nu = 0.05$ sec^{-1} is shown in Fig. 5 (a, along the x axis, b, along the y axis).

The present study will permit development of practical recommendations for carrying out ice-breaking tasks by the resonant method realized with an air cushion vessel or other periodically varying load.

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